On the distribution of linear combinations of chi-square random variables

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Abstract

The distribution of linear combinations of independent chi-square random variables is intimately related with the distribution of quadratic forms in normal random variables [1, 6, 7, 8, 9, 10, 11, 13, 14] and thus it also appears as the limit distribution of quadratic forms in non-normal random variables. As such, this distribution has been studied by many authors [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. However, there is still much room left for improvement, since while some simpler approximations do not yield sufficiently good results, other approximations which show a better performance are sometimes too complicated to be implemented in practical terms.

In this paper the exact distribution of linear combinations of independent chi-square random variables is obtained, for some particular cases, in a closed finite highly manageable form, while for the general case a near-exact approximation [3] is obtained, which is able to yield very manageable and well-performing approximations.

Keywords

Characteristic function, Gamma distribution, Generalized integer gamma distribution, Generalized near-integer gamma distribution, Mixtures.

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