

Equivalence of linear models under changes to data, design matrix, or covariance structure

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Abstract

For the mixed linear model, there is a collection of results giving conditions under which fixed parameter estimates, and/or random parameter predictors remain unchanged. Some of these results were initially developed for models with only fixed parameters, others include situations where at least some parameters are random. These equivalence results cover a range of situations - the covariance structure of error processes, design matrices, and even data may be altered. Covariance structure changes have a broad range, from conditions under which ordinary least squares estimates (OLSE) are best linear unbiased estimates (BLUE) ([9], [2]), to conditions for two sets of BLUEs and/or two sets of BLUPs to be equivalent ([10, 11], [1], [6, 7]). Changes in design structure link to adding or deleting regressors or parameters ([5]). Data changes are related to data cloning techniques ([3]), and to adding new observations ([8], [4]). These types of model modification will be discussed and various possible applications will be outlined.

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