# Variance components estimability in multilevel models with block circular symmetric covariance structure

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**Abstract**

The multilevel model with the block circular symmetric covariance structure is considered. We established the spectral properties of this patterned covariance matrix. It has been shown that the explicit maximum likelihood estimators (MLEs) of variance-covariance components do not exist in this model, unless we put restrictions on the parameter space.

It is shown that by putting restrictions on the spectrum of the block circular covariance matrices, some natural reparameterization conditions (e.g. sum-to-zero) are derived. Sufficient conditions of obtaining explicit estimators for variance-covariance components are presented. Different restricted models are discussed in order to obtain explicit estimators, get interpretable model reparameterizations and keep invariant properties of the block circular symmetric covariance structure.

In the class of restricted models, it gives us the flexibility to choose the reasonable constraints among them according to different data, which is quite advantageous.

## Keywords

Circular block symmetry, restricted model, explicit maximum likelihood estimator, variance components.

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