Weighting, model transformation, and design optimality

John P. Morgan and J. W. Stallings

Virginia Polytechnic Institute and State University, Blacksburg, Virginia, United States

Abstract

Traditional design optimality criteria place equal emphasis on estimable functions of model parameters. Use of weighted criteria allows experiments to be designed so to place increased emphasis on estimation of those functions of the parameters that are of greater interest. Here design weighting is investigated for the linear model $y = A_d\tau + L\beta + e$ in which A_d (whose column space contains the all-one vector) is the design matrix to be selected, the parameters of interest are τ, the matrix L is fixed by the experimental setup, and β is comprised of nuisance parameters including an intercept. If C_d is the information matrix for estimation of τ, then $C_{dW} = W^{-1/2}C_dW^{-1/2}$ is a weighted information matrix that for any conventional criterion Φ induces a weighted criterion Φ_W via $\Phi_W(C_d) = \Phi(C_{dW})$. The weight matrix W can be any symmetric, positive definite matrix. Among the results established are: (i) for any desired assignment of (positive) weights to any full rank set of linearly independent, estimable functions of τ there is a corresponding weight matrix W; (ii) every admissible design is weighted E-optimal with respect to some weighting; (iii) optimal design for a reparameterized model is equivalent to weighted optimality for the original model. Result (iii) demonstrates, for instance, why orthogonal arrays need not be optimal fractions under a baseline parametrization (see [2]). Families of weight matrices W are explored according to features they encompass. Among these families are the diagonal weight matrices employed in [1].

Keywords

Design admissibility, Design optimality, Weighted optimality criterion.

References

[1] Morgan, J.P. and X. Wang (2010). Weighted optimality in designed experimentation. *J. Amer. Statist. Assoc. 105*, 1566–1580.

[2] Mukerjee, R. and B. Tang (2012). Optimal fractions of two-level factorials under a baseline parameterization. *Biometrika 99*, 71–84.