Partial orders on matrices and the column space decompositions

K. Manjunatha Prasad

Manipal University, INDIA

Abstract

In literature, we have several partial orders on subclasses of rectangular matrices of same size and some which are dominated by known "Minus Partial Order". Star partial order ([3]) on rectangular matrices of size, Sharp order ([6]) on class of square matrices of same size and of index one, and the Core partial order ([2]) are such partial orders dominated by minus partial order to name a few. It is well known that the $m \times n$ matrices B and $A - B$ decomposes the given matrix A under minus partial order (i.e., $B, A - B \leq^- A$) is equivalent to say that the column spaces of B and $A - B$ decomposes the column space of the matrix A (i.e., $\mathcal{C}(B) \oplus \mathcal{C}(A - B) = \mathcal{C}(A)$). The same is true for the row spaces. In fact, there is one to one correspondence between matrix decompositions with reference to minus partial order, column space decompositions and row space decompositions. The characterization of the partial orders such as star partial order and sharp order involve both column space and row space of given matrices. In fact, matrix decomposition $A = B + C$ with reference to star partial order corresponds to decomposition of column space and row space of A orthogonally and similarly other matrix partial orders are characterized by the typical characteristic decompositions of the column space and row spaces. Even while studying the shorted matrices (see [1], [5] and [10]) involves both row space and column spaces of given matrices. Now in the light one to one correspondence between column space decompositions and row space decompositions, we characterize the partial orders with reference to column space decomposition alone. Also, it results in having a new definition of shorted matrix with reference to various partial orders i.e., only with reference to the decomposition of column space decomposition.

Keywords

Matrix partial order, Minus partial order, Star partial order, Sharp partial order, Shorted matrix.

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