

On exact and approximate simultaneous confidence regions for parameters in normal linear model with two variance components

Viktor Witkovský¹ and Júlia Volaufová²

¹*Slovak Academy of Sciences, Bratislava, Slovakia*

²*LSUHSC School of Public Health, New Orleans, USA*

Abstract

We consider normal linear regression model with two variance-covariance components

$$Y \sim N_n(X\beta, \sigma^2 V(\lambda)),$$

where X is known ($n \times p$) matrix, $\beta \in R^p$ is unknown vector of parameters and $\sigma^2 V(\lambda) = \sigma^2(I_n + \lambda V)$ is the variance-covariance matrix, with known n.n.d. matrix V , which depends on unknown parameters $\sigma^2 > 0$ and $\lambda \geq 0$.

We will present a brief overview the standard LRT/RLRT test statistics and will present the form and properties of their exact and/or approximate distributions under null hypothesis, see e.g. [1, 2], which could be used for construction of the simultaneous confidence regions for some combinations of the parameters $\theta, \lambda, \sigma^2$, where $\theta = H'\beta$, H being a known matrix such that $R(H) \subseteq R(X')$, based on inverting the exact (restricted) likelihood ratio tests of the following null hypotheses:

$$H_0 : \theta = \theta_0 \text{ and } \lambda = \lambda_0 \quad (1)$$

$$H_0 : \theta = \theta_0 \text{ and } \lambda = \lambda_0 \text{ and } \sigma^2 = \sigma_0^2 \quad (2)$$

$$H_0 : \lambda = \lambda_0 \quad (3)$$

$$H_0 : \lambda = \lambda_0 \text{ and } \sigma^2 = \sigma_0^2. \quad (4)$$

Keywords

Linear regression model with two variance components, Exact likelihood ratio test, Simultaneous confidence regions.

References

- [1] Crainiceanu, C.M. and D. Ruppert (2004). Likelihood ratio tests in linear mixed models with one variance component. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 66, 165–185.
- [2] Volaufová, J. and V. Witkovský (2012). On exact inference in linear models with two variance-covariance components. Submitted to *Tatra Mt. Math. Publ.* Proceedings from Probatat 2011.